Unit 4

Division of Whole Numbers

The 20-question Diagnostic Test for Division of Whole Numbers, in multiple-choice format, consists of four parts: multiplication facts, two-digit by one-digit division, three- or four-digit by one-digit division, and dividing by a two-digit number. The test allows you to pinpoint specific skills and concepts that require more student work. For information on how to use this test to help identify specific student error patterns, see pages 114 through 116.

Students should be encouraged to use estimation to check to see if their answers are reasonable. Reproducible lessons for estimating quotients using front-end estimation and for estimating using compatible numbers (for all operations) are on pages 175 and 176.

Pages 173 through 181 provide instructional games and follow-up activities (reproducible) to support division and multiplication concepts.

Name ______________________________ Date ______________ Class ______________

Diagnostic Test
Division of Whole Numbers

Multiple Choice: Circle the correct answer. If your answer is not given, circle *Not here.*

Part 1

1.	$48 \div 6 = \square$	**A** 6	**B** 7	**C** 8	**D** 9	**E** Not here
2.	$72 \div 9 = \square$	**A** 6	**B** 7	**C** 8	**D** 9	**E** Not here
3.	$0 \div 6 = \square$	**A** 0	**B** 1	**C** 6	**D** 60	**E** Not here
4.	$3 \div \square = 1$	**A** 1	**B** 2	**C** 3	**D** 4	**E** Not here
5.	$6 = \square \div 9$	**A** 3	**B** 6	**C** 36	**D** 54	**E** Not here

Part 2

6.	$7\overline{)5\ 9}$	**A** 7 R7	**B** 7 R10	**C** 8	**D** 8 R3	**E** Not here
7.	$3\overline{)1\ 6}$	**A** 5	**B** 5 R1	**C** 32	**D** 50 R1	**E** Not here
8.	$2\overline{)3\ 7}$	**A** 13	**B** 18 R1	**C** 81 R1	**D** 153 R1	**E** Not here
9.	$9\overline{)7\ 9}$	**A** 8	**B** 8 R7	**C** 9 R2	**D** 9 R9	**E** Not here
10.	$4\overline{)8\ 4}$	**A** 2	**B** 12	**C** 20	**D** 21	**E** Not here

Part 3

11. $2\overline{)7\ 1\ 2}$

A 301
B 311
C 356
D 3,501 R1
E Not here

12. $4\overline{)8\ 0\ 7}$

A 21 R3
B 102 R3
C 201
D 201 R3
E Not here

13. $6\overline{)5\ 1\ 2}$

A 80 R5
B 85 R2
C 163
D 850 R2
E Not here

14. $7\overline{)5,\ 7\ 4\ 2}$

A 82 R2
B 806
C 820 R2
D 8,200 R2
E Not here

15. $8\overline{)3,\ 9\ 0\ 4}$ **A** 488 **B** 884 **C** 2,102 **D** 4,880 **E** Not here

Part 4

16. 8,000 ÷ 20 = ☐ **A** 4 **B** 40 **C** 400 **D** 4,000 **E** Not here

17. $12\overline{)4\ 0\ 8}$

A 30
B 31
C 34
D 43
E Not here

18. $15\overline{)6,\ 1\ 2\ 9}$

A 48 R9
B 408 R9
C 612 R9
D 4,080 R9
E Not here

19. $24\overline{)1,\ 3\ 8\ 0}$

A 50
B 57 R12
C 69
D 690
E Not here

20. $45\overline{)4,\ 8\ 0\ 9}$

A 16 R39
B 106 R39
C 120 R9
D 1,202 R1
E Not here

ITEM ANALYSIS FOR DIAGNOSTIC TEST
Division of Whole Numbers

Using the Item Analysis Table

- The correct answer for each item on the Diagnostic Test is indicated by a ✓ in the Item Analysis Table on page 116.
- Each *incorrect* answer choice is keyed to a specific error pattern and corresponding Intervention Activity found on pages 117 through 134. Because each item on the Diagnostic Test is an item that is analyzed in one of the error patterns, teachers may be able to use the Intervention Activities with identical problems that students may have missed on the test.
- Students should be encouraged to circle *Not here* if their obtained answer is not one of the given answer choices. Although *Not here* is never a correct answer on the Diagnostic Test, the use of this answer choice should aid in the diagnostic process. The intention is that students who do not see their obtained answer among the choices will select *Not here* rather than guess at one of the other choices. This should strengthen the likelihood that students who select an incorrect answer choice actually made the error associated with an error pattern.
- The Item Analysis Table should only be used as a *guide.* Although many errors are procedural in nature, others may be due to an incorrect recall of facts or carelessness. A diagnostic test is just one of many tools that should be considered when assessing student work and making prescriptive decisions. Before being certain that a student has a misconception about a procedure or concept, further analysis may be needed (see below). This is especially true for students who frequently select *Not here* as an answer choice.
- A set of practice exercises, keyed to each of the four parts of the Diagnostic Test, is provided on page 135. Because the four parts of the set of practice exercises match the four parts on the Diagnostic Test, the set of practice exercises could be used as a posttest.

Using Teacher-Directed Questioning and Journaling

Discussions and observations should be used to help distinguish misconceptions about concepts and procedures (which often are discovered by examining error patterns) from student carelessness or lack of fact recall. Any discussion of errors should be done in a *positive* manner—with the clear purpose being to get inside student thinking. The Intervention Activities are replete with teacher-directed questioning, frequently asking students to *explain their reasoning.* Students should also be asked to *write* about their thinking as they work through an algorithm—and, when alternative algorithms are used, explain why they may prefer one algorithm over another. You may also want students to write word problems based on *division*—and then explain why division can be used to solve them. This would be a good time to discuss with students the various actions and problem structures for division (see pages 21–23).

Additional Resources for Division

- A lesson on using front-end estimation is provided on page 175; a lesson on using compatible numbers for estimation (for all operations) is on page 176. These lessons may be used at any point in the instructional process. When students engage in estimation activities, they should discuss why they believe a computed answer may or may not be reasonable. Students should also compare and contrast estimation strategies.
- Instructional games and follow-up activities designed to promote division concepts are provided on pages 173 through 181. This material may be used at any point in the instructional process. The games *Balance the Number Sentence!* and *Target Math* may be used as vehicles for observing student behavior in trial-and-error thinking, computation, and problem solving. The follow-up activity *Abbott and Costello's Number* Non*sense* provides a fun way to examine the meaning of division while uncovering some humorous errors.

ITEM ANALYSIS TABLE

The correct answer for each item on the Diagnostic Test is indicated by a ✓ in this table.

	Item	Answer Choices A	B	C	D	Topic	Practice Exercises
	1	Error 1a	Error 1a	✓	Error 1a	Division facts	Part 1, p. 135
	2	Error 1a	Error 1a	✓	Error 1a	Division facts	Part 1, p. 135
	3	✓	Error 1d	Error 1d	Error 1d	Division facts	Part 1, p. 135
	4	Error 1c	Error 1c	✓	Error 1c	Division facts	Part 1, p. 135
Part 1	5	Error 1a	Error 1a	Error 1a	✓	Division facts	Part 1, p. 135
	6	Error 2	Error 2	Error 2 or 5	✓	2-digit by 1-digit division	Part 2, p. 135
	7	Error 2 or 5	✓	Error 3	Error 8	2-digit by 1-digit division	Part 2, p. 135
	8	Error 3	✓	Error 4a	Error 4b	2-digit by 1-digit division	Part 2, p. 135
	9	Error 2 or 5	✓	Error 2	Error 2	2-digit by 1-digit division	Part 2, p. 135
Part 2	10	Errors 1c & 4a	Error 4a	Error 1c	✓	2-digit by 1-digit division	Part 2, p. 135
	11	Error 3 or 5	Error 6	✓	Error 4b	3- or 4-digit by 1-digit division	Part 3, p. 135
	12	Error 7	Error 4a	Error 3 or 5	✓	3- or 4-digit by 1-digit division	Part 3, p. 135
	13	Error 6	✓	Error 3	Error 8	3- or 4-digit by 1-digit division	Part 3, p. 135
	14	Error 7	Error 5	✓	Error 8	3- or 4-digit by 1-digit division	Part 3, p. 135
Part 3	15	✓	Error 4a	Error 3	Error 8	3- or 4-digit by 1-digit division	Part 3, p. 135
	16	Error 7	Error 7	✓	Error 8	Dividing by a 2-digit number	Part 4, p. 135
	17	Error 5	Error 6	✓	Error 4a	Dividing by a 2-digit number	Part 4, p. 135
	18	Error 7	✓	Error 9a	Error 8	Dividing by a 2-digit number	Part 4, p. 135
	19	Error 5	✓	Error 9a	Error 9b	Dividing by a 2-digit number	Part 4, p. 135
Part 4	20	Error 7	✓	Error 9a	Error 9b	Dividing by a 2-digit number	Part 4, p. 135

ERROR PATTERNS & INTERVENTION ACTIVITIES

Division of Whole Numbers

Error Pattern 1

Error Pattern 1a: Some students have difficulty recalling basic division facts (through 81 ÷ 9). A lack of understanding of the *do/undo* (inverse) relationship between multiplication and division is often a contributing factor to making these errors.

Error Pattern 1b: When dividing by 1, some students record 1 for the quotient. (For 3 ÷ 1, they record 1 for the quotient.)

Error Pattern 1c: Some students are unaware that any *nonzero* number divided by itself is 1. (For 3 ÷ 3, they may record 0 or some other value for the quotient.)

Error Pattern 1d: Some students are unaware that 0 divided by any *nonzero* number is 0. (For 0 ÷ 3, they may record 3 or some other value for the quotient.) It should be noted that some students are unaware that *0 cannot be a divisor*—and that is why "nonzero" is included in the statements of Error Patterns 1c and 1d.

Intervention

Have students "think multiplication" when they do division by finding the missing part of a division sentence. For example, for 48 ÷ 6 = □, have students think (from right to left), "*What* times 6 makes 48?" If students cannot recall that *8* × 6 = 48, have them make or use a multiplication table to help them recall the facts. (A blank table and a completed table are provided on pages 190–191.)

Students could find 6 in the shaded column on the left in the table, then slide across (along dashed arrow) to 48, and finally slide up to see that 8 (circled in the top row) answers "*What* times 6 makes 48?" Repeat the above for 48 ÷ 8 = □. The solid arrows in the table show that 6 answers "*What* times 8 makes 48?"

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Repeat with division sentences where the missing part is *not* on the right-hand side of the equal sign, such as $56 \div \square = 8$ or $\square \div 9 = 7$. Such sentences encourage students to think of the equal sign in terms of *equality* and *balance* (rather than of "find the answer").

As you relate division facts to the multiplication facts students already know, bring out the idea of fact families. Provide a multiplication sentence and then have students write the three corresponding related sentences. For $8 \times 9 = 72$, students should write all four facts in the family as shown below.

$8 \times 9 = 72$	$9 \times 8 = 72$
$72 \div 9 = 8$	$72 \div 8 = 9$

Error Patterns 1b through 1d: Give each student or group of students three small objects and three paper cups. **Ask,** "How could you use your objects and cups to model $3 \div 3$? What quotient do you get?" (Sample: Equally place the three objects into the three cups. You end up with one object in each cup. So, $3 \div 3 = 1$.) **Ask,** "How could you use your objects and cups to model $3 \div 1$? What quotient do you get?" (Sample: Place all three objects into one cup. Because you end up with three objects in the cup, $3 \div 1 = 3$.) Now **ask,** "Suppose you have zero objects and three cups. How many objects can you put in each cup?" (0 objects.) **Ask,** "What is the quotient $0 \div 3$?" (0.) Finally, write $3 \div 0$ on the board. **Ask,** "Can you divide three objects into zero cups? Explain." (No. Sample: There are no cups in which to put the objects, so this cannot be done.)

Ask students to perform each of the division exercises discussed above ($3 \div 3$, $3 \div 1$, $0 \div 3$, and $3 \div 0$) on a calculator. **Ask,** "What does your calculator show when you try to divide by 0?" (Sample: You get an error message.) Emphasize that division by 0 is impossible.

Using "Do/Undo" Operations to Show That Division by 0 Is Impossible

To provide further amplification that you cannot divide by 0, you may want to use this mathematical reasoning argument based on "do/undo" (inverse) operations. Remind students that because $15 \div 5 = 3$, we know that $3 \times 5 = 15$. Advise students that we will use do/undo operations to address the following two cases: Case 1, dividing a nonzero number by 0, and Case 2, dividing 0 by 0.

Case 1: Use do/undo operations to find $\square$ in $15 \div 0 = \square$.

Because $15 \div 0 = \square$, we know that $\square \times 0 = 15$. But we know that *no number* times 0 is equal to 15. So there is *no solution.*

You cannot divide a nonzero number by 0.

Case 2: Use do/undo operations to find ☐ in 0 ÷ 0 = ☐. (Note: It is important to address this case because some students who understand the results in Case 1 may view 0 ÷ 0 differently. They may believe that 0 ÷ 0 = 1 or 0.)

Because 0 ÷ 0 = ☐, we know that ☐ × 0 = 0. But because *any number* will satisfy ☐ × 0 = 0, *any number* would be a solution to 0 ÷ 0 = ☐. So, you could say that 0 ÷ 0 = 3, because 3 × 0 = 0. But you could also say that 0 ÷ 0 = 4, because 4 × 0 = 0. According to the *Transitive Property of Equality,* if $a = b$ and $b = c$, then $a = c$. So, if the above division results were allowed, then because

$$3 = (0 \div 0) \text{ and } (0 \div 0) = 4,$$

we would have to conclude that 3 = 4 (because both 3 and 4 would be equal to 0 ÷ 0). But we know that 3 ≠ 4! So, we say that 0 ÷ 0 is meaningless.

You cannot divide 0 by 0.

Reviewing Division of 0 by a Nonzero Number: After a discussion on division by 0, some students may become confused about dividing 0 by a nonzero number. So, it is instructive to remind students that 0 divided by a nonzero number is equal to 0. Have students find ☐ in 0 ÷ 5 = ☐. **Ask,** "Because ☐ × 5 = 0, what do we know about the value of ☐? Explain." (☐ = 0. When the product of two numbers is 0, at least one of the numbers must be 0.)

A nonzero number divided by 0 is equal to 0.

Instructional Game: Balance the ×/÷ Number Sentence!
(See pages 154–155 for game instructions. See pages 177–178 for game pieces. Students work in groups of 2 or 3.)

This domino-type game promotes memorization of the facts while having students use trial-and-error thinking to balance multiplication and division number sentences. Students use playing cards as "dominoes" to match either a number sentence with its solution or a solution with its number sentence. (Shown below, the card with the white 8 was placed next to the card with 32 ÷ ☐ = 4 because 8 makes that sentence true.)

☐ ÷ 9 = 5	**8**	**32 ÷ ☐ = 4**	**5**

Most of the game cards involve the use of basic facts. However, advise students that those cards that show two-digit computation can be solved using mathematical reasoning rather than by performing actual computations. For example, to solve 11 × 46 = ☐ × 11, students can simply use the *Commutative Property of Multiplication* to determine that ☐ = 46.

Error Pattern 2

Some students do not recognize "near facts" and are unable to do them mentally. Near facts are close to the basic facts, but have a nonzero remainder (as in 59 ÷ 7). Students may make any of the errors shown below. Some errors are made due to a lack of understanding of the possible values of a remainder in division.

$$\begin{array}{r} 7 \text{ R7 or R10} \\ 7\overline{)5\,9} \end{array}$$

The student selects 49, the multiple of 7 right before the desired multiple (56). The student then counts up to determine the remainder to either 56 (to obtain R7) or to 59 (to obtain R10). The student may not be aware that the possible remainders when you divide by 7 are *less than* 7.

$$\begin{array}{r} 8 \\ 7\overline{)5\,9} \end{array}$$

The student determines the correct quotient, but omits the remainder.

$$\begin{array}{r} 9 \text{ R4 or R7} \\ 7\overline{)5\,9} \end{array}$$

The student selects 63, the multiple of 7 right after 56, and determines the remainder by counting back. The student may count back to 59 (to obtain R4), or back to 56 (to obtain R7).

Intervention

For $7\overline{)59}$, display 59 small objects. **Ask,** "You would like to form as many groups of 7 objects each as possible. How many groups of 7 can you make?" (8 groups.) **Ask,** "What multiplication fact shows how many objects are in the 8 groups?" (7 × 8 = 56.)

Now, have students look at a multiplication table. **Ask,** "What times 7 gets you an answer that is *as close to 59 as possible—without going over 59?* What product in the table are you using?" (8; 56.) **Ask,** "How much is left over?" (3.) Explain that the amount left over is called the *remainder,* and the answer is written *8 R3.*

×	0	1	2	3	4	5	6	7	8	9
7	0	7	14	21	28	35	42	49	$\boxed{56}$	63

Now display 56 objects. **Ask,** "When you divide the 56 objects into 7 equal groups, how many are left over?" (0 objects.) **Ask,** "Suppose we have 57 objects to equally divide into 7 groups. How many will be left over?" (1 object.) **Ask,** "Why is there a (nonzero) remainder when you divide 57 by 7?" (Sample: 57 is not divisible by 7.) Repeat for 58 objects, 59 objects, all the way through 62 objects. **Ask,** "What happens when you divide 63 objects into 7 equal groups?" (Sample: There will be 9 in each group with a remainder of 0.) **Ask,** "What is the greatest possible remainder when you divide by 7?" (6.) **Ask,** "How does a remainder in any division exercise compare with the divisor in the exercise? Explain." (Sample: The remainder is always less than the divisor. If the remainder were equal to or greater than the divisor, there would be enough left over to increase the quotient.)

For purposes of teacher edification, it should be noted that many educators avoid the use of the "R" notation for remainders in equations (as in 59 ÷ 7 = 8 R3). They believe the R notation is essentially intended for use only with the division algorithm (as in $7\overline{)59}$). One reason why the R notation with equations is avoided is because multiple unequal division problems have the same answer when the answers are given with the R notation. Note that another division problem that has 8 R3 as its answer is 35 ÷ 4. However, according to the *Transitive Property of Equality,*

if **59 ÷ 7** = 8 R3 and 8 R3 = **35 ÷ 4**, then **59 ÷ 7 = 35 ÷ 4**.

But 59 ÷ 7 = 8.43 (to the nearest hundredth) and 35 ÷ 4 = 8.75. So, 59 ÷ 7 and 35 ÷ 4 are *not* equal. To avoid using an equal sign with the R notation, some educators use an arrow and record the problem as 59 ÷ 7 → 8 R3. Use your discretion as to how you prefer your students to show such division results.

Error Pattern 3

Some students compare a one-digit divisor with each digit of the dividend—and divide the larger number by the smaller number. They ignore any remainders and record no numerals below the dividend. Essentially, the student views each digit of the dividend separately—and performs a division with that digit and the divisor.

6 ÷ 3   3 ÷ 3   9 ÷ 3

$\begin{array}{r} 213 \\ 3\overline{)639} \end{array}$

3 ÷ 1   6 ÷ 3

$\begin{array}{r} 32 \\ 3\overline{)16} \end{array}$

3 ÷ 2   7 ÷ 2

$\begin{array}{r} 13 \\ 2\overline{)37} \end{array}$

7 ÷ 3   9 ÷ 7   7 ÷ 1

$\begin{array}{r} 217 \\ 7\overline{)391} \end{array}$

Note: In the first example, although the correct answer is given, the fact that no work is shown is a clue that perhaps the student has a misconception about the algorithm. A discussion with the student would be helpful in determining the student's thinking.

Intervention

Estimation: Determining If an Answer Is Reasonable

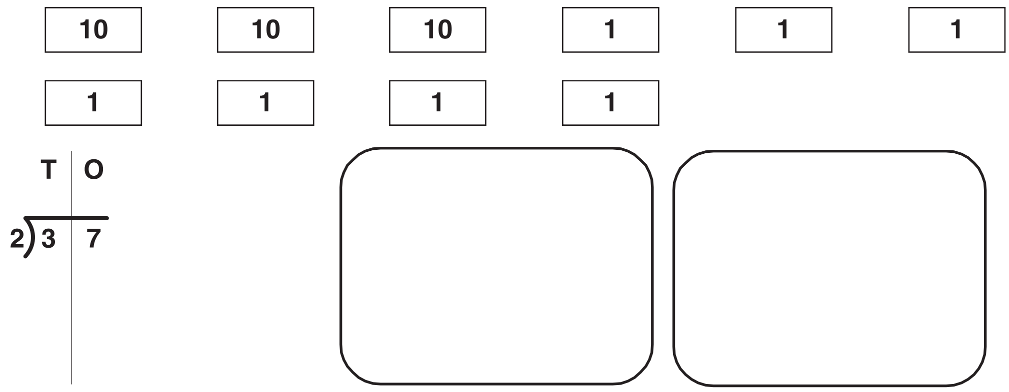
All students, especially those who obtain unreasonable answers, should be encouraged to use estimation either before or after computing. For 16 ÷ 3 = 32, **ask,** "Is it reasonable that 16 objects divided into 3 equal groups would give you 32 objects per group? Explain." (No. Sample: 32 × 3 = 96. This is far greater than the number of objects with which we started.) For 391 ÷ 7 = 217, **ask,** "Why is the quotient 217 not reasonable?" (Sample: If you round 217 to the nearest 100, you get 200. The product of 7 and 200 is 1,400. This is far greater than the dividend.) **Ask,** "How would you estimate 391 ÷ 7?" (Sample: Think of 391 as being about 350. Because 35 ÷ 7 = 5, 350 ÷ 7 = 50. So the answer should be close to 50.)

Some students may benefit from using *front-end estimation* to determine if a quotient is reasonable. The lesson "Using Front-End Estimation to Check for Reasonableness: Division" (page 175) teaches students how to use this strategy. A lesson on using *compatible numbers* to make estimates (for all operations) is on page 176. This strategy is especially effective for division—and is used in the estimate explained earlier for 391 ÷ 7.

Although estimation provides a good vehicle to determine whether or not an answer is reasonable, often a division result may be "reasonable" but incorrect. As such, students should be encouraged to check their answers by multiplying the quotient by the divisor and adding any remainder. If that result is equal to the dividend, the answer is correct.

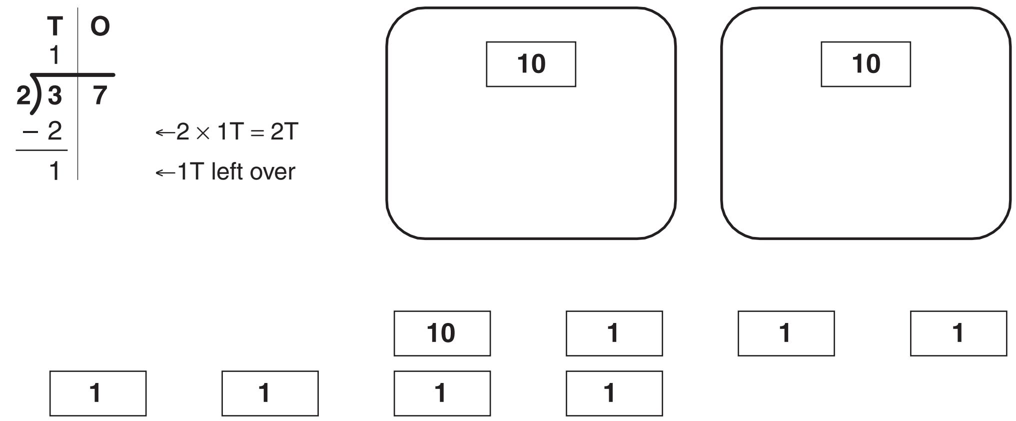
Intervention: Using Play Money or Base-Ten Blocks

Supply play money (or base-ten blocks) and a division exercise on a place-value grid. (See page 184 for play money and page 187 for grids.) To divide 37 by 2, have students display $37 and draw 2 large rings to represent 2 groups as shown.



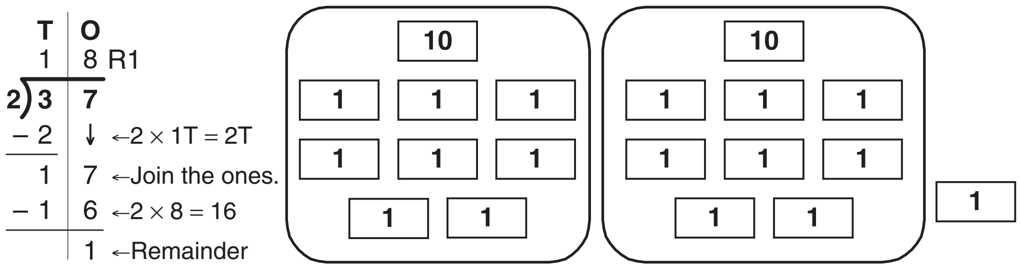
Guide students in interpreting $2\overline{)37}$ as putting $37 into 2 equal groups. **Ask,** "Are there enough ten-dollar bills to put 1 in each group?" (Yes.) **Ask,** "So, how many digits will be in the quotient?" (2 digits.) **Ask,** "Are there enough ten-dollar bills left to put another ten-dollar bill in each group?" (No.) **Ask,** "What is an estimate for the quotient?" (Sample: The quotient will be between 10 and 20.) It should be noted that such questions help focus students' attention on an estimate for the entire quotient—and not just on the first digit.

Have students put 1 ten-dollar bill in each group and record 1 in the tens position of the exercise. To show how much money was placed in the rings, instruct them to multiply the 1 ten in each ring by 2 (the number of rings) as shown. Then have them subtract the tens to show that there is 1 ten left—along with the 7 ones.



Guide students in trading the 1 ten-dollar bill that is left over for 10 one-dollar bills. Have students join those bills with the original 7 one-dollar bills, giving 17 to put in the 2 groups. Relate that to "bring down the 7" in the algorithm. Advise students that we must now put *as many one-dollar bills in each group as possible.*

Ask, "What is the greatest number of one-dollar bills you can put in each ring?" (8 one-dollar bills.) Have students place the 8 one-dollar bills in each ring and have them record 8 in the ones column of the exercise. **Ask,** "How much money is left over?" ($1.) Advise students to record R1 in the exercise.



Follow-Up Activity: Abbott and Costello's Number *Non*sense
(See pages 179–180 for the activity. Students work as an entire class, in pairs, or in small groups.)

This activity is based on an Abbott and Costello comedy routine where they attempt to perform a division computation (from their movie *In the Navy,* Gottlieb, Lubin, Horman, & Grant, 1941). In this routine, Lou Costello attempts to divide 28 by 7 but makes a series of errors. Bud Abbott tries to correct the errors by showing how to check the division—first using multiplication, then using addition. But each of Bud's attempts is countered by Lou's continued errors. In the activity, students are asked to explain the errors that were made and explain the rationale behind using multiplication and addition to check the work.

Error Pattern 4

Error Pattern 4a: Some students record the tens digit of the quotient in the ones place and the ones digit of the quotient in the tens place. The student perhaps erroneously transfers to division what is done in addition, subtraction, and multiplication—namely, recording the answer from right to left.

$$\begin{array}{r} 12 \\ 4\overline{)84} \\ \underline{-8} \\ 4 \\ \underline{-4} \\ 0 \end{array} \qquad \begin{array}{r} 81 \text{ R1} \\ 2\overline{)37} \\ \underline{-2} \\ 17 \\ \underline{-16} \\ 1 \end{array} \qquad \begin{array}{r} 05 \text{ R1} \\ 7\overline{)351} \\ \underline{-35} \\ 1 \\ \underline{-0} \\ 1 \end{array} \qquad \begin{array}{r} 43 \\ 12\overline{)408} \\ \underline{-36} \\ 48 \\ \underline{-48} \\ 0 \end{array}$$

Error Pattern 4b: Some students first divide the ones digit of the dividend, then the tens digit, and finally the hundreds digit—rather than beginning with the leftmost digit.

$$\begin{array}{r} 21 \\ 4\overline{)84} \\ \underline{-4} \\ 8 \\ \underline{-8} \\ 0 \end{array} \qquad \begin{array}{r} 153 \text{ R1} \\ 2\overline{)37} \\ \underline{-6} \\ 31 \\ \underline{-30} \\ 1 \end{array} \qquad \begin{array}{r} 470 \text{ R4} \\ 7\overline{)351} \\ \underline{-0} \\ 51 \\ \underline{-49} \\ 3\;\;2 \\ \underline{-2\;\;8} \\ 4 \end{array} \qquad \begin{array}{r} 400 \\ 12\overline{)408} \\ \underline{-0} \\ 08 \\ \underline{-0} \\ 4\;\;8 \\ \underline{-4\;\;8} \\ 0 \end{array}$$

Note: In the first example, the correct answer is given. However, observation of the work reveals that the student first divided the ones digit by 4 and then the tens digit by 4. A discussion with the student would be helpful in determining the student's thinking.

In the other examples, this misconception leads to incorrect answers. For example, in the second example, the student first divides 7 by 2, and records 3 in the ones position. The 3 from the dividend is then "brought down" and combined with the 1 that was left over to form 31. Then 31 is divided by 2, yielding 15 in the quotient.

Intervention

Use the Intervention Activity for Error Pattern 3, emphasizing that we record the digits in the quotient from *left to right* as you divide in each place-value position from *left to right.* Encourage students to use estimation to determine if their answers are reasonable. To help students focus on dividing from left to right, have them use an index card to cover the digits in the columns that are *not* under consideration in a given step as shown at the right for 351 ÷ 7.

$$\begin{array}{r|l} \textbf{H} & \textbf{T} \\ & 5 \\ \hline 7\overline{)\mathbf{3}} & \mathbf{5} \\ -3 & 5 \\ \hline & \end{array}$$